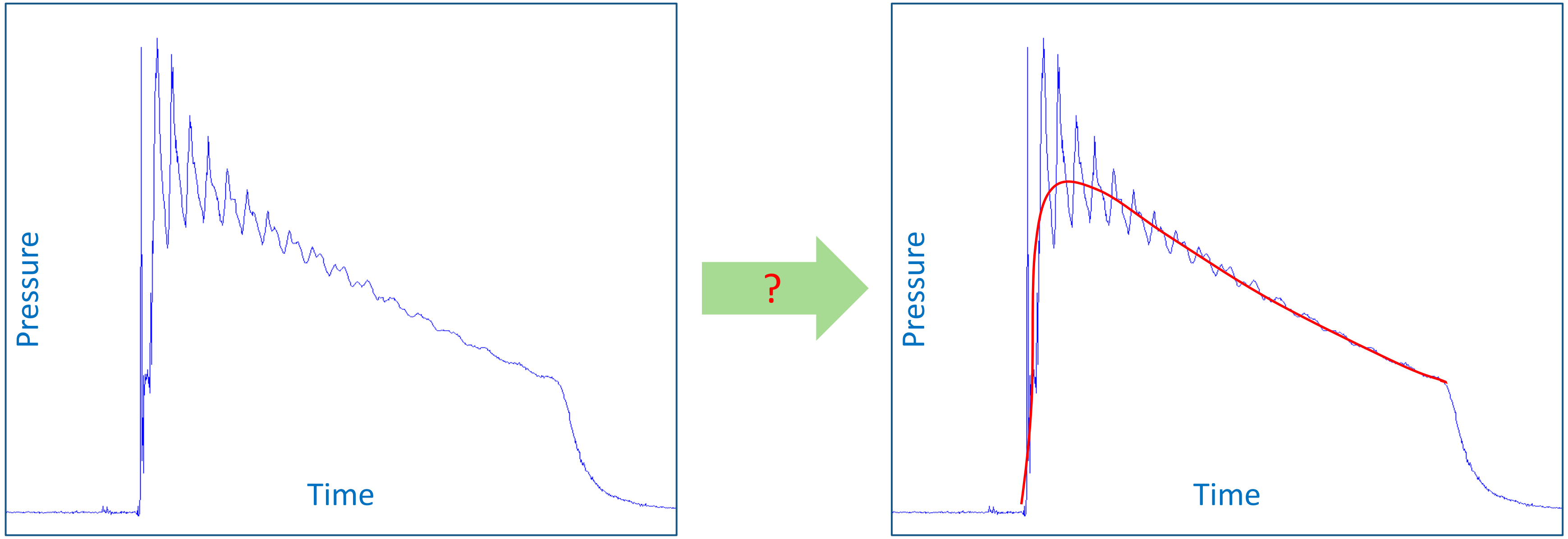



# UNSTEADY GAS-DYNAMICS EFFECTS IN CAD/PAD

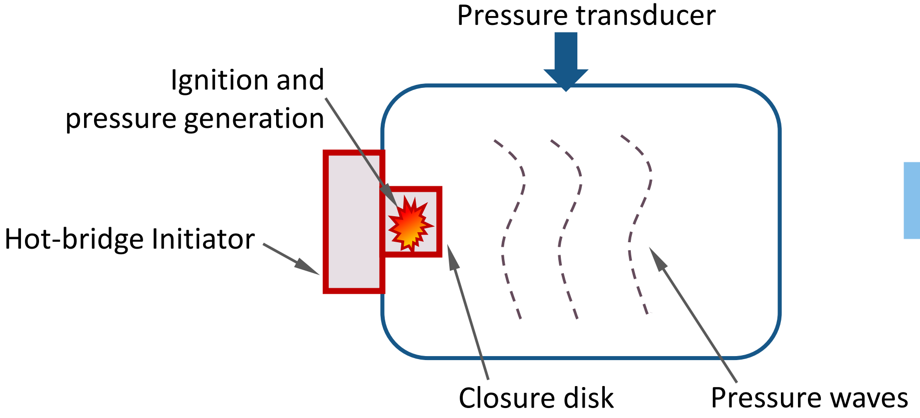
Hobin Lee

Chemring Energetic Devices
hlee@ced.us.com, (630) 788-6526

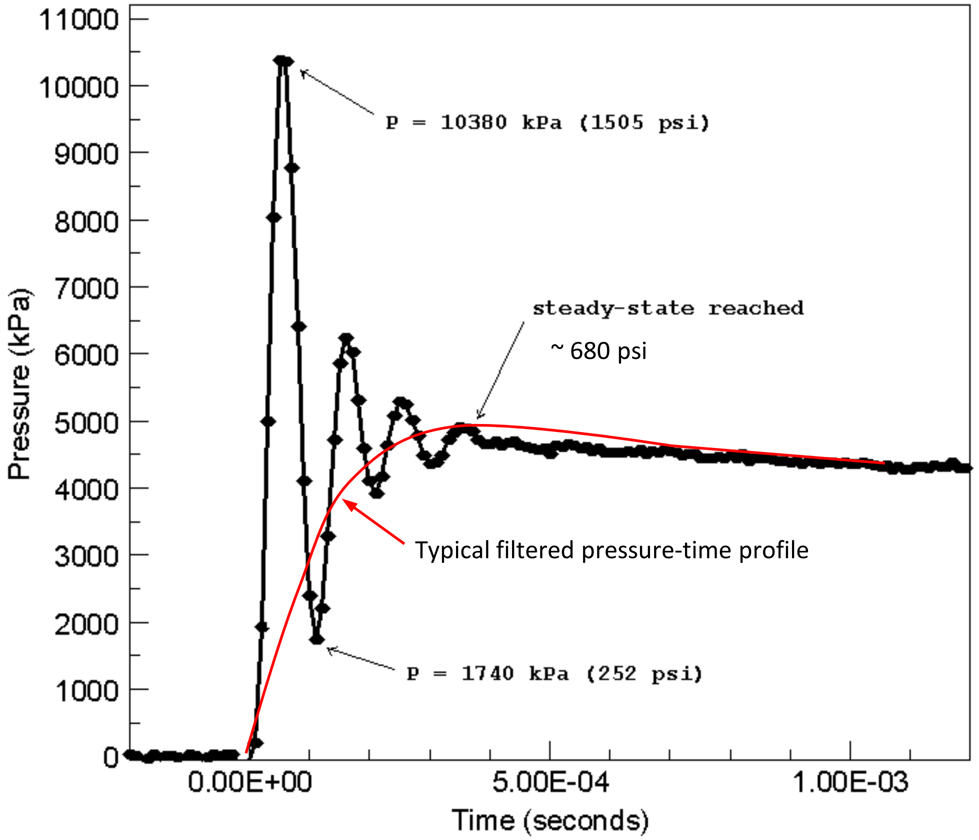
2024 CAD/PAD TECHNICAL EXCHANGE WORKSHOP, NSWC IHD

# WHAT IS MEANT BY UNSTEADY GAS-DYNAMICS EFFECTS?

**Notional Closed-bomb Firing of an Initiator**




**Unfiltered Pressure-time the Initiator**

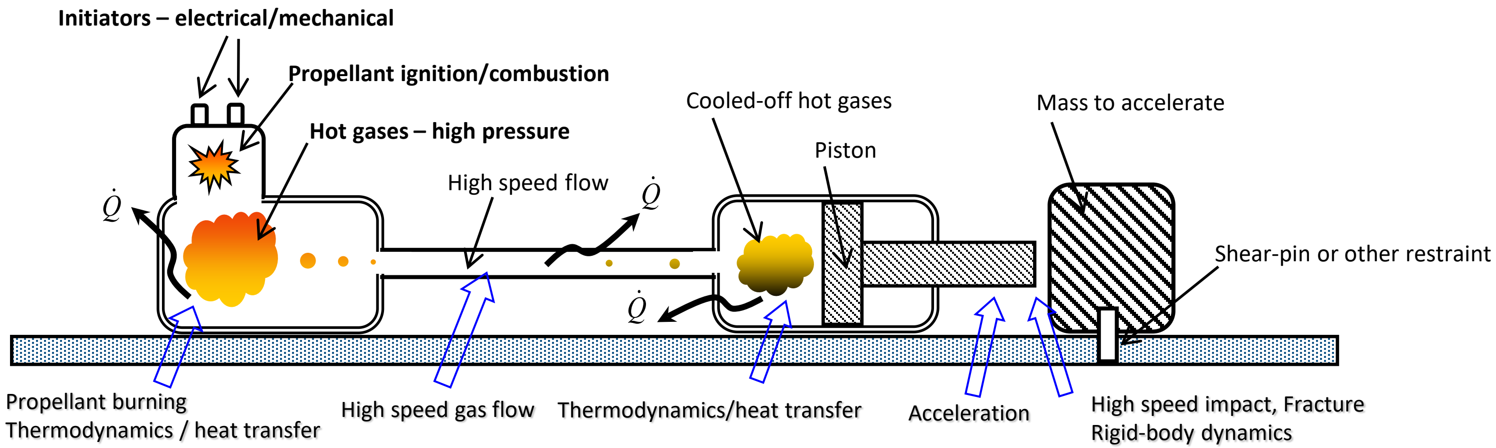



- Rapid generation of pressure by Initiator and the delay in rupturing of closure disk produce high pressure pulse
- Complex interaction between the pressure waves while the bulk pressure inside closed bomb is increasing due to the propellant burning
- Often the high frequency fluctuations are filtered during testing or post data processing
- The high frequency pressure waves may or may not contribute significantly to the overall system performance

# MODELS FOR ESTIMATING CAD/PAD PERFORMANCE

## Reality is often *complex*, but may be understood through *simplified models*

**CAD/PAD Interior Ballistics in a Nutshell**

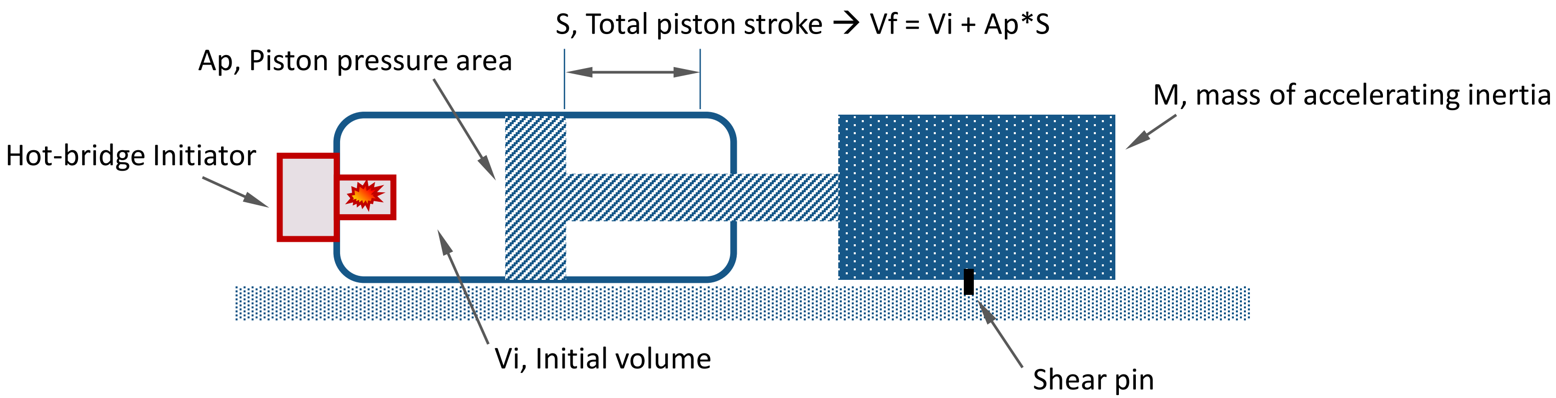



- CAD/PAD performance is a collection of multi-physics events (often occurring simultaneously)
- Assumptions can be made to simplify each of the physical events
- Integrity of the model depends on the reasonableness of the assumptions
- Testing is irreplaceable to verify the reasonableness of the assumptions



Chemring
Innovating to protect

# 1ST ORDER (BACK OF THE ENVELOPE) MODEL FOR A CAD




- Assume instantaneous burn for the charge → Pmax occurs before movement → Pmax occurs in Vi
- Assume negligible resistance and no heat transfer → isentropic work

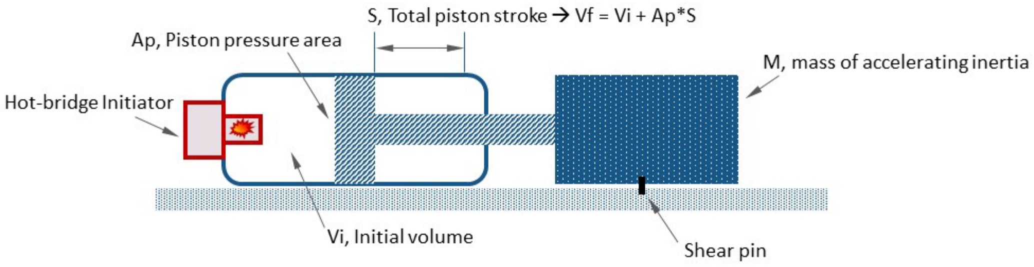
- $pV^{\gamma} = constant$
- $W_{isen} = \int pdV = \frac{p_2 V_2 - p_1 V_1}{1-\gamma}$



Chemring
Innovating to protect

# ONE LEVEL DEEPER MODEL: QUASI-EQUILIBRIUM INTERNAL BALLISTICS

- Major assumption: all thermodynamic properties are spatially homogeneous for each time step. For example,
- P and T are P(t) and T(t) only – has no spatial gradation




**Conservation of Mass**

$$\frac{d}{dt}\int_V \rho_k dV = -\int_S \rho_k v_k dS + \sum_{j=1}^{r}\int_V v_{kj} J_j dV \;\Rightarrow\; \left.\frac{dm}{dt}\right|_V = \dot{m}_p - \cancel{\dot{m}_e}$$

$$\dot{m}_p = \frac{d}{dt}\left(m_{p_o}\sum_j l_j \lambda^j\right) \qquad \frac{d\lambda}{dt} = c_o \cdot P^n$$

Production of gas is a function of intrinsic burn-rate and burning surface regression geometry

**Conservation of Energy**

$$\frac{d}{dt}\int_V \rho e dV = -\int_S (\rho e v + P\cdot v + \dot{q}) dS \;\Rightarrow\; \left.\frac{d}{dt}(m\cdot e)\right|_V = -PA_p \cdot v + \dot{Q} \qquad de = c_P dT$$

The rate of change in energy in a control volume is the rate of work done to it and heat transfer into it.

**Equation of State**

$$P(V-\overline{\eta}) = m_p \overline{R} T$$

Correction factor added for non-ideal gases

*Noble-Abel equation of state*

**Equation of Motion**

$$\sum_i F_i = \frac{d}{dt}(M_B \cdot v_B) \;\Rightarrow\; \frac{d^2x}{dt^2} = (P\cdot A_P - F_{ext})/\,M_B$$

# ONE-DIMENSIONAL UNSTEADY GAS DYNAMICS MODEL

## But what about the presence of pressure fluctuations?

- Major assumption: all thermodynamic properties are function of time and one spatial dimension. For example,
- P and T are P(t,x) and T(t,x)

**Lagrangian Coordinate**

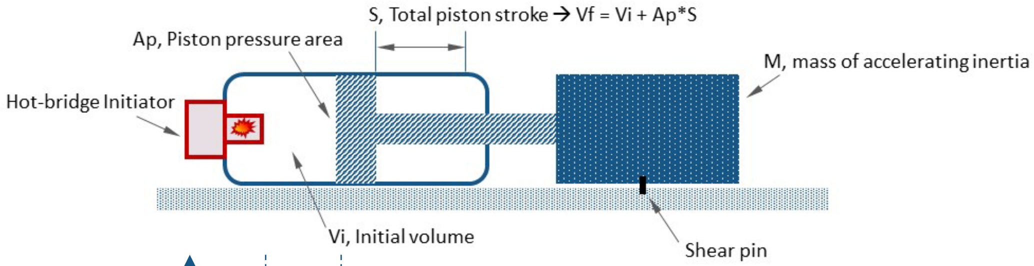
$$\xi = \int^{x} \rho(x')dx'$$

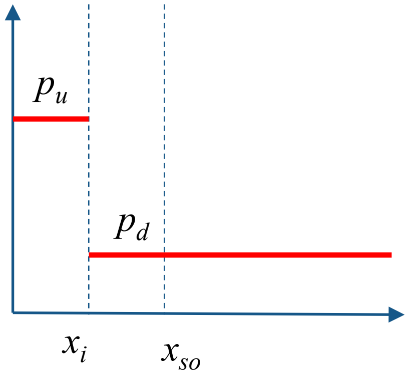
$$\frac{\partial \upsilon}{\partial t} - \frac{\partial u}{\partial \xi} = 0$$ Continuity

$$\frac{\partial u}{\partial t} + \frac{\partial p}{\partial \xi} = -\alpha u^2$$ Momentum

$$\frac{\partial e}{\partial t} + \frac{\partial}{\partial \xi}(u \cdot p) = \dot{q}$$ Energy

$$p = \rho(\gamma - 1)(e - u^2/2)$$ Energy/pressure




*$x_i$ can be chosen to dial up or down $p_u$ and effectively the shock strength*

**Initial Condition**

$$\begin{cases} \upsilon(0,\xi) = \upsilon_o(x) \\ P(0,\xi) = P_o(x) \\ u(0,\xi) = 0 \end{cases}$$

$$\text{where} \quad P_o, \upsilon_o(x) = \begin{cases} P_u, \upsilon_u & x \le x_i \\ P_d, \upsilon_d & x > x_i \end{cases}$$

**Boundary Condition**

$$\begin{cases} \upsilon, P(t,0) = \text{Rigid wall shock reflection condition} \\ u(t,0) = 0 \\ u(t,\xi_R) = \dfrac{A_p}{M_p} \int P_d dt \end{cases}$$

*Unsteady gas dynamics aspect can be incorporated into the quasi-equilibrium model for a fuller simulation, including the finite burning of the propellants*

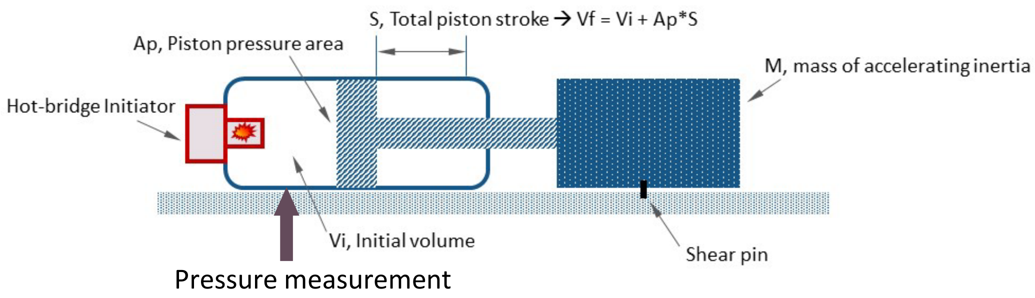
# COMPARISON OF MODELS

## What level of fidelity is sufficient for a "good" working model?

- Depends… what fidelity is desired or required?
- However, it is important to know each model's assumptions and its limitations and *when the assumptions will significantly affect* the model's integrity.

## So, when does the "unsteadiness" or pressure fluctuations become significant player in CAD/PAD?

- Compare the results of two models: Quasi-equilibrium and Unsteady gas dynamics models <u>in a simple CAD example</u>

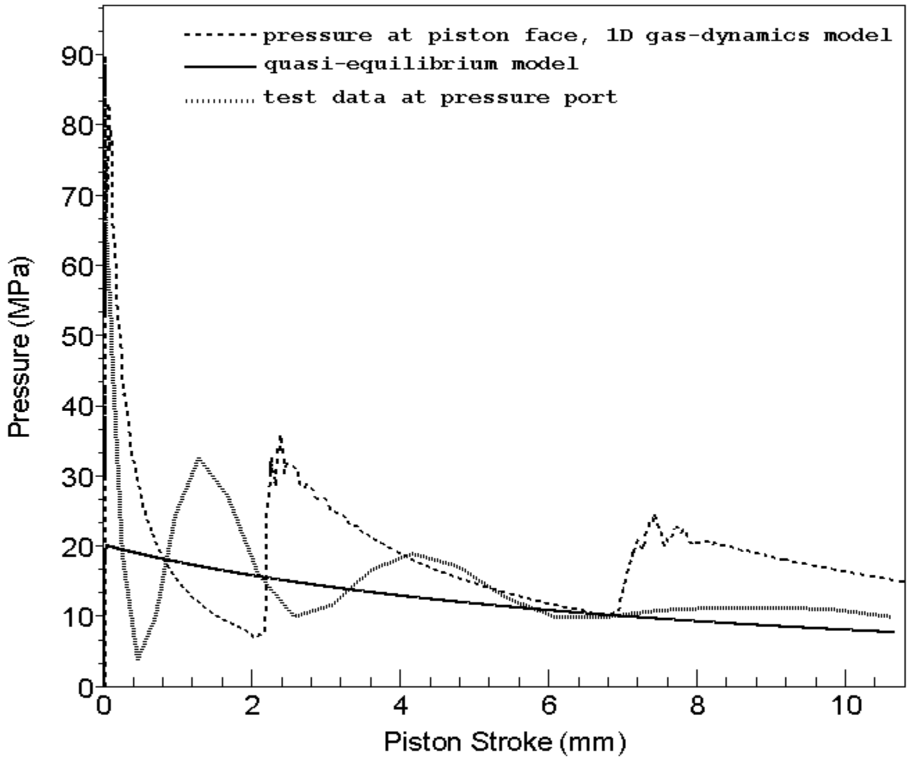


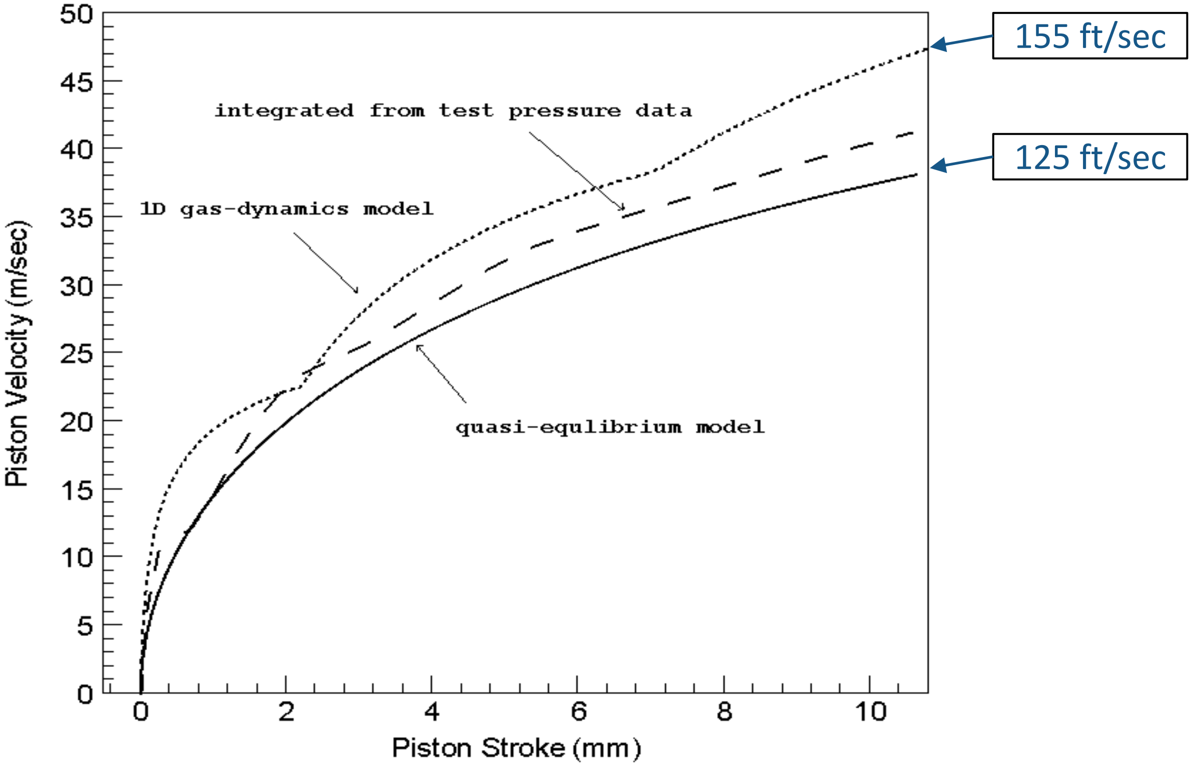

| CAD Dimensions/Characteristics | |
|---|---|
| Initiator Output in 10cc (psi) | 210 |
| Total inertia or moving mass (lbs) | 0.03 |
| Piston Area (sq-in) | 0.11 |
| Pressure chamber ID (in) | 0.37 |
| Total stroke (in) | 0.42 |
| Initial Volume (in3) | 0.03 |



Chemring
Innovating to protect

# COMPARISON OF MODELS CONTINUED




- Internal pressure predictions are significantly different between the two models – as expected
- The final mass velocity predictions also differ noticeably – 35% difference in kinetic energy at the end of stroke



Chemring
Innovating to protect

# A CRITICAL PARAMETER FOR UNSTEADY EFFECTS

## Define a parameter that can indicate whether the unsteady effects will or will not be significant

- A ratio of a piston dynamics characteristics time vs. gas-dynamics characteristics time: $\tau_C = \tau_D / \tau_G$

### Gas Dynamics Time

$$\tau_G^* = 5(L_T / c_s)$$

$$\left(\frac{c_s}{c_r}\right)^2 = 1 + \frac{2(\gamma-1)}{(\gamma+1)^2}\left[\gamma\left(\frac{W_s}{c_r}\right)^2 - \left(\frac{c_r}{W_s}\right)^2 - (\gamma-1)\right]$$

$$\frac{p_s}{p_r} = 1 + \frac{2\gamma}{\gamma+1}\left[\left(\frac{W_s}{c_r}\right)^2 - 1\right]$$

$$\frac{p_r}{p_u} = \frac{p_r}{p_s}\left(1 - \frac{\gamma-1}{2\gamma}\frac{\left(\frac{p_s}{p_r}-1\right)}{\sqrt{1+\frac{\gamma+1}{2\gamma}\left(\frac{p_s}{p_r}-1\right)}}\right)^{\frac{2\gamma}{\gamma-1}}$$

$L_T$ = $S$ + Initial length between Initiator and Piston face Total Stroke
$c_s$ = Speed of sound of initial shock
$c_r$ = Speed of sound of undisturbed medium downstream of initial shock
$W_s$ = Shock speed
$P_s$ = Shock pressure
$P_r$ = Pressure of undisturbed medium downstream of initial shock
$P_u$ = Pressure in initiator just prior to disk closure rupture
$\gamma$ = ratio of coefficients of heat, $c_p/c_v$

### Piston Dynamics Time

$$\tau_D = \sqrt{S/a}$$

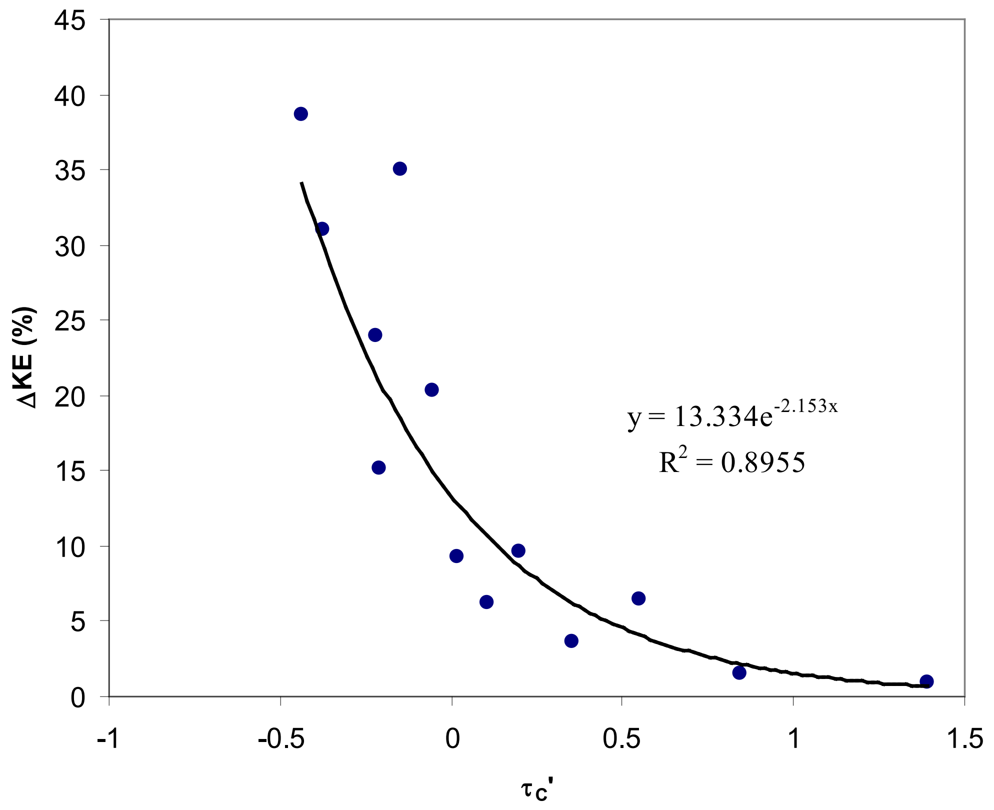
$$a = A_p(P_{CB}V_{CB}/V_o)/M_p$$

$S$ = Total Stroke
$A_P$ = Piston Area
$P_{CB}$ = Pmax in a closed bomb volume of $V_{CB}$
$V_o$ = Initial device internal volume
$M_P$ = total moving mass

* The factor 5 is applied by considering a flow discharging through a duct from a reservoir. In such a case, a steady flow condition is approximately reached when the value *ct/L* is about 5

# A CRITICAL PARAMETER FOR UNSTEADY EFFECTS CONTINUED

| Case | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $A_p$ | 0.023 | 0.023 | 0.023 | 0.023 | 0.023 | 0.023 | 0.023 | 0.023 | 0.049 | 0.049 | 0.049 | 0.049 |
| $D_i$ | 0.172 | 0.172 | 0.172 | 0.172 | 0.172 | 0.172 | 0.172 | 0.172 | 0.25 | 0.25 | 0.25 | 0.25 |
| $M_p$ | 0.028 | 0.028 | 0.015 | 0.01 | 0.005 | 0.028 | 0.015 | 0.015 | 0.028 | 0.01 | 0.028 | 0.012 |
| $x_{so}$ | 1.98 | 0.60 | 1.98 | 1.98 | 1.98 | 1.98 | 1.98 | 1.98 | 1.98 | 1.98 | 0.6 | 0.6 |
| Total stroke | 0.42 | 0.42 | 0.42 | 0.42 | 0.42 | 0.42 | 0.22 | 0.42 | 0.22 | 0.22 | 0.42 | 0.42 |
| Initiator $P_{CB}$ in 10cc | 210 | 210 | 210 | 210 | 210 | 650 | 850 | 850 | 650 | 650 | 650 | 650 |
| **$\tau_C$** | 1.85 | 2.39 | 1.35 | 1.10 | 0.78 | 1.20 | 0.62 | 0.79 | 0.95 | 0.56 | 1.55 | 1.01 |
| **% $\Delta$KE** | 1.5 | 0.9 | 3.7 | 6.3 | 24.0 | 9.6 | 31.0 | 15.1 | 20.3 | 38.7 | 6.5 | 9.3 |




- **Kinetic energy difference exponentially increases beyond $\tau_c' < 0$, $\tau_c' = \tau_c - 1$**
- **Unsteady or pressure fluctuation effects become increasingly significant when the characteristics time for gas dynamics is smaller or shorter than the inertial or piston dynamics characteristics time**



Chemring
Innovating to protect

# CONCLUDING REMARKS AND CIBAC

- Physics in CAD/PAD are complex
- Models that illustrate the important aspects of the complex physics are possible
- Appropriate assumptions can be made for simpler models
- It's important to understand the limitations of the model due to its assumptions and when the limitations will become non-negligible factors to the desired accuracy of the model
- ***CIBAC***, Chemring Internal Ballistics Analysis Code, is a proven tool for addressing various assumptions
    - Physics based model with modules that can be easily tailored for various scenarios or problems to solve
    - Can quickly assess the effects of assumptions
    - Can quickly provide high fidelity CAD/PAD performance estimations

The discussions on the unsteady gas-dynamics effects are based on the contents of previously published paper titled, "Unsteady Gasdynamics Effects in Pyrotechnic Actuators", H. Lee, *AIAA Journal of Spacecraft and Rockets*, Vol. 41, No. 5, September-October, 2004.