SOLUTIONS

REACTOR DYNAMICS PRACTICE PROBLEMS FOR TECHNICAL MAJORS

The DOE Fundamentals Handbook: *Nuclear Physics and Reactor Theory, Volume 2 of 2*, provides a general overview of some of the important concepts of reactor theory. It is written to cover a generic commercial nuclear reactor whose primary purpose is electric power generation, and not a Naval reactor whose main purpose is propulsion. Consequently, not all of the concepts presented and terminology used will be identical to what you will see at Nuclear Power School (NPS). Nevertheless, the DOE Handbook provides a good background and preparation for the Reactor Dynamics subject at NPS. The time and effort you devote to this prep material will make your learning process easier while in class at NPS.

Topic 1 – "Neutron Life Cycle" Questions

1. Define the following terms:

a. Infinite multiplication factor ($\mathbf{k}_\infty$): The ratio of the neutrons produced by fission in one generation to the number of neutrons lost through absorption in the preceding generation.

b. Effective multiplication factor ($\mathbf{k}_{\mathbf{eff}}$): The ratio of the neutrons produced by fission in one generation to the number of neutrons lost through absorption and leakage in the preceding generation.

c. Subcritical: If neutron production is less than absorption and leakage, the reactor is called subcritical. In a *subcritical* reactor, $\mathbf{k}_{\mathbf{eff}} < \mathbf{1}$, and the neutron flux decreases each generation.

d. Critical: The condition where the neutron chain reaction is self-sustaining and the neutron population is neither increasing nor decreasing. In a *critical* reactor, $\mathbf{k}_{\mathbf{eff}} = \mathbf{1}$.

e. Supercritical: If neutron production is greater than absorption and leakage, the reactor is called supercritical. In a *supercritical* reactor, $\mathbf{k}_{\mathbf{eff}} > \mathbf{1}$, and the neutron flux increases each generation.

2. Name and define each factor in the six factor formula using the ratio of the number of neutrons present at different points in the neutron life cycle.

$$\text{Reproduction Factor} = \eta = \frac{\text{number of fast neutrons produced by thermal fission}}{\text{number of thermal neutrons absorbed in the fuel}}$$

$$\text{Thermal Utilization Factor} = f = \frac{\text{number of thermal neutorns absorbed in the fuel}}{\text{number of thermal neutrons absorbed in all reactor materials}}$$

$$\text{Resonance Escape Probability} = p = \frac{\text{number of neutrons that reach thermal energy}}{\text{number of fast neutrons that start to slow down}}$$

$$\text{Fast Fission Factor} = \varepsilon = \frac{\text{number of fast neutrons produced by all fissions}}{\text{number of fast neutrons produced by thermal fissions}}$$

$$\text{Fast Non-Leakage Probability} = \mathcal{L}_f = \frac{\text{number of fast neutrons that do not leak from reactor}}{\text{number of fast neutrons produced by all fissions}}$$

$$\text{Thermal Non-Leakage Probability} = \mathcal{L}_{th} = \frac{\text{number of thermal neutrons that do not leak from reactor}}{\text{number of neutrons that reach thermal energies}}$$

NOTE: These practice problems as well as NPS use a "th" subscript for the thermal non-leakage probability rather than just a "t" as in the DOE Handbook.

3. Calculate the thermal utilization factor, f, for a homogeneous reactor. The macroscopic absorption cross section of the fuel is 0.2028 cm^{-1}, the macroscopic absorption cross section of the moderator is 0.0110 cm^{-1}, and the macroscopic absorption cross section of the poison is 0.0218 cm^{-1}.

$$f = \frac{\Sigma_a^U}{\Sigma_a^U + \Sigma_a^m + \Sigma_a^p}$$

$$f = \frac{0.2028\ \text{cm}^{-1}}{0.2028\ \text{cm}^{-1} + 0.0110\ \text{cm}^{-1} + 0.0218\ \text{cm}^{-1}}$$

$$f = 0.861\ .$$

4. Twenty thousand (20,000) neutrons exist at the beginning of a generation. The values for each factor of the six factor formula are given below. Calculate the number of neutrons that exist at the points in the neutron life cycle as listed below and the single value of the effective multiplication factor.

a. Number of neutrons that exist after fast fission
b. Number of neutrons that start to slow down in the reactor
c. Number of neutrons that reach thermal energies
d. Number of thermal neutrons that are absorbed in the reactor
e. Number of thermal neutrons absorbed in the fuel
f. Number of neutrons produced from thermal fission.

$\boldsymbol{\varepsilon = 1.028} \quad \boldsymbol{\mathcal{L}_f = 0.886} \quad \mathbf{f = 0.754}$

$\mathbf{p = 0.813} \quad \boldsymbol{\mathcal{L}_{th} = 0.955} \quad \boldsymbol{\eta = 2.021}$

a. $N = N_o \varepsilon = 20{,}560$
b. $N = N_o \varepsilon \mathcal{L}_f = 18{,}216$
c. $N = N_o \varepsilon \mathcal{L}_f\, p = 14{,}810$
d. $N = N_o \varepsilon \mathcal{L}_f\, p\, \mathcal{L}_{th} = 14{,}143$
e. $N = N_o \varepsilon \mathcal{L}_f\, p\, \mathcal{L}_{th}\, f = 10{,}664$
f. $N = N_o \varepsilon \mathcal{L}_f\, p \mathcal{L}_{th}\, f\ \eta = 21{,}552.$

$$\begin{aligned} k_{eff} &= \varepsilon\, \mathcal{L}_f\, p\, \mathcal{L}_{th} f\, \eta \\ &= (1.028)(0.886)(0.813)(0.955)(0.754)(2.021) \\ &= 1.0776\ . \end{aligned}$$

As a check of our work, note that

$$\begin{aligned} N_f &= k_{eff} N_o \\ &= (1.0776)(20{,}000) \\ &= 21{,}552 \text{ after one generation.} \end{aligned}$$

5. Explain the effect that temperature changes will have on the following factors:

a. Thermal utilization factor

When the temperature rises, the water moderator expands and a significant amount of it will be forced out of the reactor core. This means that N_m, the number of moderator atoms per cm^3, will be reduced, making it less likely for a neutron to be absorbed by a moderator atom. This reduction in N_m results in an increase in thermal utilization as moderator temperature increases because a neutron now has a better chance of hitting a fuel atom.

b. Fast non-leakage probability
As coolant temperature rises, the coolant expands. The density of the moderator is lower; therefore, neutrons must travel farther while slowing down. This effect increases the probability of leakage and thus decreases the non-leakage probability.

c. Resonance escape probability
As coolant temperature rises, the coolant expands. The density of the moderator is lower; therefore, neutrons must travel farther while slowing down. This effect increases the probability of a fast neutron being absorbed and thus decreases the resonance escape probability.

d. Reproduction factor
Reproduction factor is dependent on fuel properties alone and is unaffected by changes in coolant temperature.

Topic 2 – "Reactivity" Questions

1. Using the data from Problem 4 in the previous topic, find the reactivity.

$$\begin{aligned}\rho &= \frac{k_{eff}-1}{k_{eff}}\\ &= \frac{1.0776-1}{1.0776}\\ &= 0.0720 \text{ or } 720\text{x}10^{-4} \text{ reactivity units.}\end{aligned}$$

NOTE: NPS uses the symbol (δk) *for reactivity instead of the DOE* ρ*, and NPS the symbol* $(\Delta\delta k)$ *for change in reactivity instead of the DOE* $\Delta\rho$*.*

2. If a neutron population doubles in 50 generations, calculate the reactivity.

$$\frac{N_{50}}{N_o} = (k_{eff})^{50} = 2 \Rightarrow k_{eff} = e^{\left(\frac{\ell n2}{50}\right)} = 1.014$$

$$\rho = \frac{k_{eff}-1}{k_{eff}} = \frac{1.014-1}{1.014} = 0.0138 \text{ or } 138\text{x}10^{-4} \text{ reactivity units.}$$

3. Can the reactivity and the effective multiplication factor be positive? Negative? Zero?

Reactivity can be positive, negative, or zero. However, k_{eff} can <u>only</u> be a positive value.

4. For a reactivity of $40x10^{-4}$, calculate the effective neutron multiplication factor.

$$\begin{aligned} k_{eff} &= \frac{1}{1-\rho} \\ &= \frac{1}{1-40x10^{-4}} \\ &= 1.0040\ . \end{aligned}$$

5. List at least three parameters that affect the core's reactivity.

Fuel concentration
Poison concentration
Temperature
Pressure
(there are additional correct answers).

6. If the temperature coefficient of reactivity is $-0.90x10^{-4}\ \frac{\text{reactivity units}}{^{o}F}$, and the moderator temperature increases by 10 degrees F, will the reactivity increase, decrease, or remain the same? Will the effective multiplication factor increase, decrease, or remain the same?

Using $\Delta\rho = \alpha_T\ \Delta T$, reactivity will decrease due to the negative value of α_T,

Using $K_{eff} = \frac{1}{1-\rho}$, the value of k_{eff} will decrease.

<u>Topic 3 – "Reactivity Coefficients" Questions</u>

1. List three properties of a good moderator.

Large neutron scattering cross-section
Small neutron absorption cross-section
Large neutron energy loss per collision.

2. For an under-moderated reactor that uses ordinary water as the moderator, compare and explain the signs of the pressure and temperature coefficients of reactivity.

In under-moderated reactors, if temperature rises, the density of the moderator and moderator-to-fuel ratio will decrease, causing a drop in the effective multiplication factor and an insertion of negative reactivity. Therefore, the temperature coefficient of reactivity is negative. (continued)

If pressure rises, the density of the moderator and moderator-to-fuel ratio will increase, causing a rise in the effective multiplication factor and an insertion of positive reactivity. Therefore, the temperature coefficient of reactivity is positive.

3. Why can changes in reactivity due to pressure usually be neglected in a reactor that is cooled and moderated by subcooled water?
It can usually be neglected because a change in pressure causes a very small change in the density of a subcooled liquid. This makes the pressure coefficient of reactivity negligible compared to the temperature coefficient of reactivity.

Topics 4, 5, & 6 – "Neutron Poisons", "Xenon", and "Samarium and Other Fission Product Poisons" Questions

1. Explain the use of burnable neutron poisons in a reactor core.
Burnable neutron poisons are used in reactor cores to compensate for the excess positive reactivity of the fuel when the reactor is initially started up.

2. What is the equation for equilibrium xenon-135 concentration?

$$N_{eq}^{xe} = \frac{\left(\gamma^{I} + \gamma^{xe}\right) \Sigma_{f}^{fuel}\ \phi}{\lambda_{xe} + \sigma_{a}^{xe}\ \phi}$$

3. How does the equilibrium xenon-135 concentration vary with reactor power level?
The xenon-135 concentration increases with increasing power level in a non-linear manner. Equilibrium xenon-135 concentration reaches a maximum at a flux of about 10^{15} $\text{neutrons}/\text{cm}^2\text{-sec}$.

4. Describe how xenon-135 concentration changes following a decrease in the power level of a reactor.
After a power decrease, xenon-135 concentration will initially increase due to production by iodine decay being greater than the burnout. Xenon-135 will reach a maximum about 8 hours after the power decrease and then decrease to a new, lower equilibrium value.

5. How is samarium-149 produced and removed from the reactor core during reactor operation?
Samarium-149 is produced directly from fission and from the decay of promethium-149 during reactor operation. Samarium-149 is removed from the core by neutron absorption.

Topic 7 – "Control Rods" Questions

1. What are the qualities of a good control rod material?

A good control rod material should have a large neutron absorption cross section and should not burn out rapidly (i.e., have a long lifetime.)

2. Define Integral Control Rod Worth and Differential Control Rod Worth.

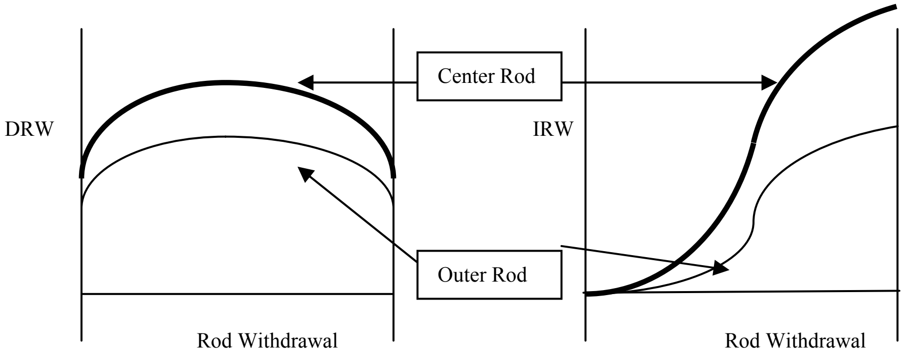
IRW – The total reactivity worth of the rod at that particular degree of rod withdrawal (rod height).

DRW – The change in reactivity to unit movement of a rod.

3. For a rod located at the center of the core, draw a sketch of Differential Rod Worth vs Rod Withdrawal and a sketch of Integral Rod Worth vs Rod Withdrawal.

See below.

4. On the same axis, complete question 3 for a rod located near the edge of the core. (Hint: for any given rod height, neutron flux is always lower at the edge than it is in the center.)



5. If a rod is currently 15-inches withdrawn, how much farther would it have to be withdrawn to add and additional 40 pcm of reactivity? Use the graphs from Example 1 on page 53 of the DOE Handbook: "Nuclear Physics and Reactor Theory, Volume 2 of 2; Module 3 – Reactor Theory (Nuclear Parameters)".

From IRW vs RH graph: RH_o = 15-inches, and RH_f is approximately 22-inches,

$\Rightarrow \Delta RH$ = 7-inches.