SOLUTIONS

MATH PRACTICE PROBLEMS FOR NON-TECHNICAL MAJORS

DOE "MATHEMATICS" VOLUME 1 of 2

FRACTIONS

Perform the indicated operations. Leave all answers in reduced fractional form.

1. $\dfrac{\dfrac{\mathbf{4}}{\mathbf{21}}}{\dfrac{\mathbf{30}}{\mathbf{84}}} + \mathbf{3}\dfrac{\mathbf{1}}{\mathbf{2}} - \dfrac{\mathbf{13}}{\mathbf{2}}$

$$= \frac{4}{21}\times\frac{84}{30}+\frac{7}{2}-\frac{13}{2} = \frac{336}{630}+\frac{7}{2}-\frac{13}{2} = \frac{8}{15}+\frac{7}{2}-\frac{13}{2} = \frac{8\times 2}{15\times 2}+\frac{7\times 15}{2\times 15}-\frac{13\times 15}{2\times 15}$$

$$= \frac{16}{30}+\frac{105}{30}-\frac{195}{30} = \frac{-74}{30} = -\frac{37}{15} = -2\frac{7}{15}$$

2. $-\mathbf{10}\dfrac{\mathbf{2}}{\mathbf{7}} + \mathbf{16}\dfrac{\mathbf{17}}{\mathbf{68}} - \mathbf{7}\dfrac{\mathbf{18}}{\mathbf{33}}$

$$= -\frac{72}{7}+\frac{1105}{68}-\frac{249}{33} = -\frac{161568}{15708}+\frac{255255}{15708}-\frac{118524}{15708} = \frac{-24837}{15708} = -1\frac{9129}{15708}\left(\frac{\frac{1}{3\cdot 17}}{\frac{1}{3\cdot 17}}\right) = -1\frac{179}{308}$$

3. $\dfrac{\mathbf{6}}{\mathbf{(x+2)(x-2)}} + \dfrac{\mathbf{2}}{\mathbf{(x-2)}}$

$$= \frac{6}{(x+2)(x-2)}+\frac{2(x+2)}{(x+2)(x-2)} = \frac{6+2x+4}{(x+2)(x-2)} = \frac{2x+10}{x^2-2x+2x-4} = \frac{2(x+5)}{x^2-4}$$

EXPONENTS

Simplify the following:

1. $\mathbf{8\cdot 3^{2x}+9^{x}}$

$$= 8\cdot\left(3^2\right)^x + 9^x = 8\cdot 9^x + 9^x = 9^x\cdot(8+1) = 9^x\cdot 9 = 9^{x+1}$$

2. $\mathbf{15^{y-1}\cdot 3^{1-y}}$

$$= 15^{y-1}\cdot 3^{-(y-1)} = \frac{15^{y-1}}{3^{(y-1)}} = \left(\frac{15}{3}\right)^{y-1} = 5^{y-1}$$

3. $\dfrac{\mathbf{x^{-4}yz^{-2}}}{\mathbf{x^{-5}y^{-3}z^{3}}}$

$= x^{-4-(-5)}y^{1-(-3)}z^{-2-3} = x^{1}y^{4}z^{-5} = \dfrac{xy^{4}}{z^{5}}$

4. $\left(\dfrac{\mathbf{x^{-1}y^{-2}-x^{-3}}}{\mathbf{x^{-3}y^{-1}-x^{-2}y^{-2}}}\right)^{-2}$

$= \left(\dfrac{x^{3}y^{2}}{x^{3}y^{2}}\cdot\dfrac{x^{-1}y^{-2}-x^{-3}}{x^{-3}y^{-1}-x^{-2}y^{-2}}\right)^{-2} = \left(\dfrac{x^{2}y^{0}-x^{0}y^{2}}{x^{0}y^{1}-x^{1}y^{0}}\right)^{-2} = \left(\dfrac{x^{2}-y^{2}}{y-x}\right)^{-2} = \left(\dfrac{y-x}{x^{2}-y^{2}}\right)^{2}$

SCIENTIFIC NOTATION

Perform the indicated operations. Place all answers in scientific notation and round to **5** significant figures.

1. $\mathbf{212755+63713473}$

$=63926228=6.3926\times10^{7}$

2. $\mathbf{.0000006876250\times7.114499}$

$=\left(6.876250\times10^{-7}\right)\left(7.114499\right)=48.92107375\times10^{-7}=4.8921\times10^{-6}$

3. $\mathbf{.0007800491\times10^{-4}\div4107063}$

$=7.800491\times10^{-8}\div4.107063\times10^{6}=1.8993\times10^{-14}$

ALGEBRAIC LAWS

Expand the following, and simplify.

1. $\mathbf{2a+5a(ab-4)-3ab(a+3)}$

$=2a+5a^{2}b-20a-3a^{2}b-9ab=2a^{2}b-9ab-18a=a\left(2ab-9b-18\right)$

2. $\dfrac{\mathbf{a\left[a+\left(b+c\right)\right]bc}}{\mathbf{abc+b^{2}c+c^{2}b}}$

$=\dfrac{a\left[abc+bbc+cbc\right]}{abc+b^{2}c+c^{2}b}=\dfrac{a\left[abc+b^{2}c+c^{2}b\right]}{abc+b^{2}c+c^{2}b}=a$

LINEAR EQUATIONS

Solve for x.

1. $\mathbf{a = \frac{b - cx}{2 - x}}$

$a(2-x) = b-cx$

$2a-ax=b-cx$

$(cx-2a)+2a-ax=b-cx+(cx-2a)$

$cx-ax=b-2a$

$x(c-a) = b-2a$

$x=\frac{b-2a}{c-a}$

2. $\mathbf{\frac{3x}{4} - \frac{1}{7}(3x + 5) = 14}$

$\frac{3x}{4} - \frac{3x}{7} - \frac{5}{7} = 14$

$\frac{3x \cdot 7}{4 \cdot 7} - \frac{3x \cdot 4}{7 \cdot 4} - \frac{5 \cdot 4}{7 \cdot 4} = 14$

$\frac{21x - 12x - 20}{28} = 14$

$9x - 20 = 14 \cdot 28$

$9x = 412$

$x = 45.778$

3. $\mathbf{\frac{1}{3x} + \frac{1}{x+2} = \frac{1}{3x^2 + 6x}}$

$\frac{1}{3x}\left(\frac{x+2}{x+2}\right) + \frac{1}{x+2}\left(\frac{3x}{3x}\right) = \frac{1}{3x^2 + 6x}$

$\frac{x+2}{3x^2 + 6x} + \frac{3x}{3x^2 + 6x} = \frac{1}{3x^2 + 6x}$

$\frac{x+2}{3x^2 + 6x}(3x^2 + 6x) + \frac{3x}{3x^2 + 6x}(3x^2 + 6x) = \frac{1}{3x^2 + 6x}(3x^2 + 6x)$

$x + 2 + 3x = 1$

$4x = -1$

$x = -\frac{1}{4}$

4. $\dfrac{2x+5}{4x+1}=\dfrac{x-2}{2x-1}$

$\dfrac{2x+5}{4x+1}\left[(2x-1)(4x+1)\right]=\dfrac{x-2}{2x-1}\left[(2x-1)(4x+1)\right]$

$(2x+5)(2x-1)=(x-2)(4x+1)$

$4x^2-2x+10x-5=4x^2+x-8x-2$

$8x-5=-7x-2$

$8x+7x=-2+5$

$15x=3$

$x=\dfrac{1}{5}$

QUADRATIC EQUATIONS

Find the roots.

1. $x^2-8x+16=0$

$(x-4)(x-4)=0$

$x=4,\ 4$

2. $13x-3=4x^2+4x$

$(13x-3)-13x+3=(4x^2+4x)-13x+3$

$0=4x^2-9x+3$

$4x^2-9x+3=0$

$x=\dfrac{-(-9)\pm\sqrt{(-9)^2-4\cdot4\cdot3}}{2\cdot4}=\dfrac{9\pm\sqrt{81-48}}{8}=\dfrac{9\pm\sqrt{33}}{8}=\dfrac{9\pm5.7446}{8}$

$x=\dfrac{14.7446}{8},\ \dfrac{3.2554}{8}=1.8431,\ 0.40693$

SIMULTANEOUS EQUATIONS

Solve the following simultaneous equations for both x and y:

1. $3y - 2x = 4, \quad y + 2x = 0$

$y = -2x$

$3(-2x)-2x=4$

$-8x=4$

$x = -\frac{1}{2}$

$y = -2\left(-\frac{1}{2}\right) = 1$

2. $y = ax + bx, \qquad y = cx + d$

$ax+bx=cx+d$

$ax+bx-cx=d$

$[a+b-c]x = d$

$x = \frac{d}{[a+b-c]}$

$y = cx + d = c\left(\frac{d}{[a+b-c]}\right) + d = \frac{cd}{a+b-c} + d$

OR

$y = ax + bx = (a+b)x = (a+b)\left(\frac{d}{[a+b-c]}\right) = \frac{(a+b)d}{[a+b-c]}$

WORD PROBLEMS

1. If a consumer is billed $210 for 1500 kilowatt-hours (KWH) of electrical use, what consumption would result in a bill of $125? Assume a direct proportion between amount of the bill and consumption.

Let x = unknown consumption in KWH for a bill of $125.
Then, the equation for the direct proportion is $125/x = $210/1500 KWH.
Alternatively, the direct proportion can be written as $125/$210 = x/1500 KWH.
Thus, x = ($125)(1500 KWH)/($210) = 892.86 kilowatt-hours.

2. A submarine leaves Pearl Harbor, heading for Bangor, Washington. At the same time, another submarine leaves San Diego, heading for Bangor. If the submarine leaving Pearl Harbor travels twice as fast as the submarine leaving San Diego, and takes 14 days to get to Bangor, how long will it take the other submarine to arrive? (For simplicity, assume that the distance from Bangor to Pearl Harbor is three times the distance from Bangor to San Diego.)

Let $d_1 =$ distance from Pearl Harbor to Bangor.

Let $d_2 =$ distance from San Diego to Bangor.

Let $t_1 =$ the time the Pearl Harbor sub takes to arrive.

Let $t_2 =$ the time the San Diego sub takes to arrive.

Let $v_1 =$ the speed of the Pearl Harbor sub.

Let $v_2 =$ the speed of the San Diego sub.

Then, $d_1 = v_1 t_1 = 3d_2, \;\; d_2 = v_2 t_2 = \left(\frac{1}{2} v_1\right) t_2$

Therefore, $v_1 t_1 = 3\left[\left(\frac{1}{2} v_1\right) t_2\right] = \frac{3}{2} v_1 t_2$

$$\frac{v_1 t_1}{v_1} = \frac{\frac{3}{2} v_1 t_2}{v_1}$$

$$t_1 = \frac{3}{2} t_2$$

$$t_2 = \frac{2}{3} t_1 = \frac{2}{3}(14\text{days}) = \frac{28}{3}\text{days} = 9\frac{1}{3}\text{days}$$

<u>LOGARITHMS</u>

Solve for x.

1. $10^x = 2$

$\log 10^x = \log 2$

$x = \log 2 = 0.30103$

2. $6^x = 4$

$\ln 6^x = \ln 4$

$x \ln 6 = \ln 4$

$x = \frac{\ln 4}{\ln 6} = .77371$

OR

$\log 6^x = \log 4$

$x \log 6 = \log 4$

$x = \frac{\log 4}{\log 6} = .77371$

3. $\mathbf{e^x = a + b}$

$\ln e^x = \ln(a+b)$ $\quad \left[\text{Note: } \ln(a+b) \neq \ln a + \ln b\right]$

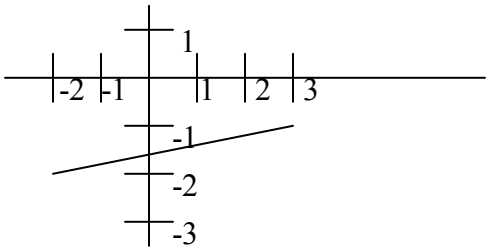
$x = \ln(a+b)$

GRAPHING

Graph the following as y=f(x).

1. $\mathbf{x = 3y + 5 \text{ for } -2 \le x \le 3}$

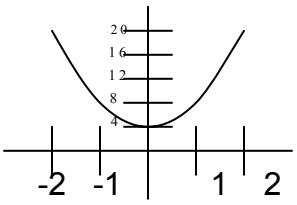
$$y = \frac{x-5}{3}$$



2. $\mathbf{y = (2x)^2 + 3^2 - 5 \text{ for } -2 \le x \le 2}$

$y = 4x^2 + 9 - 5$

$y = 4x^2 + 4$



INTERPOLATION AND EXTRAPOLATION

1. **In GRAPHING problem 1 above, extrapolate the value of x at y=0.**
   Extrapolation gives x=5. Solving using the equation, we find:
   $x = 3(0) + 5 = 5$

2. **In GRAPHING problem 2 above, interpolate the value of y at x=1.5**
   y(1)=8 and y(2)=20, so we interpolate y(1.5)=14. Using the equation we find:
   $y = 4(1.5)^2 + 4 = 13$

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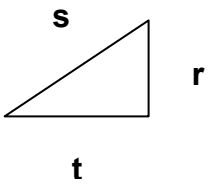
PYTHAGOREAN THEOREM

1. r = 4, s = 7, t = ?
Solve for t.



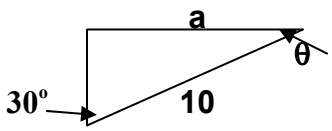
$r^2 + t^2 = s^2$

$t^2 = s^2 - r^2$

$t = \sqrt{s^2 - r^2} = \sqrt{7^2 - 4^2} = 5.7446$

TRIGONOMETRIC FUNCTIONS

1. Find a and cos θ :



$\sin 30^\circ = \frac{1}{2} = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{10}$ Note: For this angle, the opposite is side a.

However, side a is the adjacent for θ.

$\frac{1}{2} = \frac{a}{10}$

$a = \frac{10}{2} = 5$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{a}{10} = \frac{5}{10} = \frac{1}{2}$

2. What is θ above?

$\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$

RADIANS

1. Change 90° to radians.

$$90^\circ \times \frac{\pi \text{ radians}}{180^\circ} = \frac{\pi}{2} \text{radians}$$

2. Change $\frac{3\pi}{2}$ to degrees.

$$\frac{3\pi}{2} \times \frac{180^\circ}{\pi} = 270^\circ$$

IMAGINARY AND COMPLEX NUMBERS

1. Solve for the roots of y: $3y^2 = y - 2$

$$3y^2 - y + 2 = 0$$

$$y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 3 \cdot 2}}{2 \cdot 3} = \frac{1 \pm \sqrt{1-24}}{6} = \frac{1}{6} \pm \frac{\sqrt{-23}}{6} = \frac{1}{6} \pm \frac{4.7958\,i}{6}$$

2. Find $Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2} + Z_3$, where $Z_1 = 3 + 2i$, $Z_2 = 4 - i$, and $Z_3 = 2i$

$$Z_1 Z_2 = (3 + 2i)(4 - i) = 12 - 3i + 8i - 2i^2 = 12 + 5i - 2(-1) = 14 + 5i$$

$$Z_1 + Z_2 = (3 + 2i) + (4 - i) = 7 + i$$

$$\frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{14 + 5i}{7 + i}\left(\frac{7 - i}{7 - i}\right) = \frac{98 - 14i + 35i - 5i^2}{49 - 7i + 7i - i^2} = \frac{98 + 21i - 5(-1)}{49 - (-1)} = \frac{103 + 21i}{50} = \frac{103}{50} + \frac{21}{50}i$$

$$Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2} + Z_3 = \frac{103}{50} + \frac{21}{50}i + 2i = \frac{103}{50} + \frac{121}{50}i$$